Exact Travelling Wave Solutions of the Nonlinear Evolution Equations in Mathematical Physics by using Enhanced (G'/G)-Expansion Method

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Abstract *— With the help of symbolic computation software, the present paper investigates the exact travelling wave solutions from the general (2+1)-dimensional nonlinear evolution equations by using enhanced (G'/G)-expansion method. As outcomes, the utilized method is successfully employed, and newly produced some exact travelling wave solutions. The newly produced solutions have been expressed in terms of trigonometric and hyperbolic functions. The produced solutions have been checked back into their corresponding equation with the aid of symbolic computation software Maple. Among the produced solutions, some solutions have been visualized by 3D and 2D line graphs under the choice of suitable arbitrary parameters to show their physical interpretation. The produced solutions demonstrate the power of the executed method to assess the exact solutions of the nonlinear (2+1)-dimensional nonlinear evolution equations, which are realistically applicable for utilizing the nonlinear science, mathematical physics and engineering. The enhanced (G'/G)-expansion method is reliable treatment for searching essential nonlinear waves that enrich variety of dynamic models arises in engineering fields.*

Keywords — *Enhanced (G'/G)-expansion, Calogero–Bogoyavlenskii–Schiff equation, General ($2+1$)-dimensional nonlinear evolution equation, Traveling wave solutions, Hyperbolic function*

I. INTRODUCTION

In recent years, nonlinear partial differential equations (NPDEs) is widely used to describe many important phenomena and dynamic processes in various fields of science and engineering, especially in fluid mechanics, hydrodynamics, mathematical biology, diffusion process, solid state physics, plasma physics, neural physics, chemical kinetics and geo-optical fibres.

In this work, we will study the generalized ($2+1$)-dimensional nonlinear evolution equations in the form

$$u_{xt} + au_x u_{xy} + bu_{xx} u_y + u_{xxxy} = 0 \qquad (1)$$

Recently, some special cases of equation (1) have been studied by several authors [1-4]. When setting a =4 and b = 2, equation (1) becomes the (2+1)-dimensional Calogero–Bogoyavlenskii–Schiff (CBS) equation:

$$u_{xt} + 4u_x u_{xy} + 2u_{xx} u_y + u_{xxxy} = 0 \qquad (2)$$

When setting $a = -4$ and $b = -2$, equation (1) becomes the (2+1)-dimensional breaking soliton equation:

$$u_{xt} - 4u_x u_{xy} - 2u_{xx} u_y + u_{xxxy} = 0 \qquad (3)$$

When setting a = 4 and b = 4, equation (1) becomes the (2+1)-dimensional Bogoyavlenskii's breaking soliton equation:

$$u_{xt} + 4u_x u_{xy} + 4u_{xx} u_y + u_{xxxy} = 0 \qquad (4)$$

Nowadays, the exact traveling wave solution for nonlinear partial differential equations (PDEs) has been investigated by many authors. Many powerful method have been presented such as the Hirota's bilinear transformation method [5] [6], the tanh-function method [7] [8], the extended tanh-method [9] [10], modified Exp-function method [11], the(G'/G)-expansion method [12-14], asymptotic methods and nano mechanics

[15] and so on. Also, Najafi *et al.* [16] established G'/G-expansion method for (2+1)-dimensional nonlinear evolution equations. In that method, they selected (2+1)-dimensional Bogoyavlenskii's Breaking soliton and (2+1)-dimensional Calogero-Bogoyavlenskii-Schiff (CBS) equation to exemplify the effectiveness of their method.

The article is prepared as follows: In section II, the enhanced (G'/G)-expansion method has been discussed. In section III, we apply this method to the nonlinear evolution equations pointed out above. In section IV, represents results and discussion as well as comparison with related methods. Lastly, in section V, conclusions are given.

II. DESCRIPTION OF THE ENHANCED (G'/G)- EXPANSION METHOD

In this section, we will describe in details the enhanced (G'/G)-expansion method for finding traveling wave equations of nonlinear equations. Any nonlinear equation in two independent variables x and t can be expressed in following form:

$$\Re(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}.........) = 0, \tag{5}$$

Where $u(\xi) = u(x,t)$ is an unknown function, $\Re$ is a polynomial of $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. The following steps are involved in finding the solution of nonlinear equation (5) using this method.

Step 1: The given PDE (5) can be transformed into ODE using the transformation $\xi = x \pm \omega t$, where ω is the speed of traveling wave such that $\omega \in R - \{0\}$. The traveling wave transformation permits us to reduce equation (5) to the following ODE:

$$\Re(u, u', u'', ...) = 0 \tag{6}$$

where $\Re$ is a polynomial in $u(\xi)$ and its derivatives, where $u'(\xi) = \frac{du}{d\xi}$, $u''(\xi) = \frac{d^2u}{d\xi^2}$, and so on.

Step 2: Now we suppose that the equation (6) has a general solution of the form

$$u(\xi) = \sum_{i=-n}^{n} \left(\frac{a_i (G'/G)^i}{(1 + \lambda (G'/G))^i} + b_i (G'/G)^{i-1} \sqrt{\sigma \left(1 + \frac{(G'/G)^2}{\mu}\right)} \right), \tag{7}$$

subject to the condition that $G = G(\xi)$, satisfy the equation

$$G'' + \mu G = 0, \tag{8}$$

where $a_i, b_i (-n \le i \le n; n \in N)$, and λ are constant to be determined, provided that $\sigma = \pm 1$ and $\mu \ne 0$.

Step 3: The positive integer n can be determined by balancing the highest order derivatives to the highest order nonlinear terms appear in equation (5) or in equation (6). More precisely, we define the degree of $u(\xi)$ as $D(u(\xi)) = n$ which gives rise to the degree of other expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right) = n + q,\ D\left(u^p \left(\frac{d^q u}{d\xi^q}\right)^s\right), \tag{9}$$
$$= np + s(n+q)$$

Step 4: We substitute equation (7) into equation (6) and use equation (8). We then collect all the coefficient of $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)}$ together. Since equation (7) is a solution of equation (8). We can set each of the coefficient equal to zero which leads to a system of algebraic equations in terms of $a_i, b_i(-n \le i \le n; n \in N)$, λ and ω . One can solves easily these system equations using Maple.

Step 5: For $\mu < 0$ general solution of equation (2.4) gives

$$\frac{G'}{G} = \sqrt{-\mu}\tanh\left(A+\sqrt{-\mu}\ \xi\right), \tag{10}$$

and

$$\frac{G'}{G} = \sqrt{-\mu}\coth\left(A+\sqrt{-\mu}\ \xi\right), \tag{11}$$

And for $\mu > 0$, we get

$$\frac{G'}{G} = \sqrt{\mu}\tan\left(A-\sqrt{\mu}\ \xi\right), \tag{12}$$

and

$$\frac{G'}{G} = \sqrt{\mu}\cot\left(A+\sqrt{\mu}\ \xi\right), \tag{13}$$

where A is an arbitrary constant. Finally, we can construct a number of families of travelling wave solutions of equation (5) by substituting the values of $a_i, b_i(-n \le i \le n; n \in N)$, λ and ω (obtained in Step 3) and using equation (10) to equation (13) into equation (7).

III. APPLICATION OF THE METHOD

In this section, we will exert the enhanced (G'/G)-expansion method to solve the equation (1). Now using the traveling wave variable $\xi = x + y - \omega t$, and integrating with respect to ξ reduces equation (1) to the following ordinary differential equation for $u = u(\xi)$.

$$-\omega u' + \left(\frac{a+b}{2}\right)(u')^2 + u''' = 0 \tag{14}$$

where, primes denote the differentiation with regard to ξ. By balancing u''' and $(u')^2$, we obtain $N = 1$. Therefore, the enhanced (G'/G)-expansion method admits to solution of equation (5)

$$u(\xi) = a_0 + \frac{a_1(G'/G)}{1+\lambda(G'/G)} + \frac{a_{-1}(1+\lambda(G'/G))}{(G'/G)} + b_0(G'/G)^{-1}\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)}$$
$$+ b_1\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)} + b_{-1}(G'/G)^{-2}\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)},$$

where $G = G(\xi)$ satisfies equation $G'' + \mu G = 0$. Substituting equation (15) into equation (14) and using equation (8), we get a polynomial in $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)}$. Setting the coefficient of

$(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma\left(1+\frac{(G'/G)^2}{\mu}\right)}$ equal to zero, we obtain a system containing a large number of algebraic equations in terms of unknown coefficients. We have solved this system of equations using Maple 13 and obtained the following set of solutions:

Set 1: $\omega=-4\mu,\ a_0=a_0,\ a_1=0,\ a_{-1}=-\dfrac{12\mu}{(a+b)},$,

$b_0=0,\ b_1=0,\ b_{-1}=0$

Set 2: $\omega=-\mu,\ a_0=a_0,\ a_1=0,\ a_{-1}=-\dfrac{6\mu}{(a+b)},$

$b_0=\pm\dfrac{6\mu}{\sqrt{\sigma}(a+b)},\ b_1=0,\ b_{-1}=0$

Substituting Set 1-Set 2 into equation (15) along with equation (10) - equation (13); we get the following families of traveling wave solutions.

A Hyperbolic Function Solutions

When $\mu<0$, we get the following five families of hyperbolic function solutions.

Family 1:

$$u_1(\xi)=a_0-\frac{12\mu(1+\lambda\sqrt{-\mu}\tanh(A+\sqrt{-\mu}(x+y+4\mu t)))}{(a+b)\sqrt{-\mu}\tanh(A+\sqrt{-\mu}(x+y+4\mu t))}$$

$$u_2(\xi)=a_0-\frac{12\mu(1+\lambda\sqrt{-\mu}\coth(A+\sqrt{-\mu}(x+y+4\mu t)))}{(a+b)\sqrt{-\mu}\coth(A+\sqrt{-\mu}(x+y+4\mu t))}$$

where $\xi=x+y+4\mu t$,

Family 2:

$$u_{3,4}(\xi)=a_0-\frac{6\mu(1+\lambda\sqrt{-\mu}\tanh(A+\sqrt{-\mu}(x+y+\mu t)))}{(a+b)\sqrt{-\mu}\tanh(A+\sqrt{-\mu}(x+y+\mu t))}\pm\frac{6\mu\sqrt{(1-\tanh(A+\sqrt{-\mu}(x+y+\mu t))^2)}}{(a+b)\sqrt{-\mu}\tanh(A+\sqrt{-\mu}(x+y+\mu t))}$$

$$u_{5,6}(\xi)=a_0-\frac{6\mu(1+\lambda\sqrt{-\mu}\coth(A+\sqrt{-\mu}(x+y+\mu t)))}{(a+b)\sqrt{-\mu}\coth(A+\sqrt{-\mu}(x+y+\mu t))}\pm\frac{6\mu\sqrt{(1-\coth(A+\sqrt{-\mu}(x+y+\mu t))^2)}}{(a+b)\sqrt{-\mu}\coth(A+\sqrt{-\mu}(x+y+\mu t))}$$

where $\xi=x+y+\mu t$.

B Trigonometric Function Solutions

When $\mu>0$, we get the following five families of trigonometric function solutions.

Family 3:

$$u_7(\xi) = a_0 - \frac{12\sqrt{\mu}(1+\lambda\sqrt{\mu}\tan(A-\sqrt{\mu}(x+y+4\mu t)))}{(a+b)\tan(A-\sqrt{\mu}(x+y+4\mu t))}$$

$$u_8(\xi) = a_0 - \frac{12\sqrt{\mu}(1+\lambda\sqrt{\mu}\cot(A+\sqrt{\mu}(x+y+4\mu t)))}{(a+b)\cot(A+\sqrt{\mu}(x+y+4\mu t))}$$

Where $\xi = x+y+4\mu t$

Family 4:

$$u_{9,10}(\xi) = a_0 - \frac{6\sqrt{\mu}(1+\lambda\sqrt{\mu}\tan(A-\sqrt{\mu}(x+y+\mu t)))}{(a+b)\tan(A-\sqrt{\mu}(x+y+\mu t))} \pm \frac{6\sqrt{\mu}\sqrt{(1+\tan(A-\sqrt{\mu}(x+y+\mu t))^2)}}{(a+b)\tan(A-\sqrt{\mu}(x+y+\mu t))}$$

$$u_{11,12}(\xi) = a_0 - \frac{6\sqrt{\mu}(1+\lambda\sqrt{\mu}\cot(A+\sqrt{\mu}(x+y+\mu t)))}{(a+b)\cot(A+\sqrt{\mu}(x+y+\mu t))} \pm \frac{6\sqrt{\mu}\sqrt{(1+\cot(A+\sqrt{\mu}(x+y+\mu t))^2)}}{(a+b)\cot(A+\sqrt{\mu}(x+y+\mu t))}$$

where $\xi = x+y+\mu t$.

IV. RESULT AND DISCUSSION

In this section we will describe about the physical explanations and graphical representation of the general ($2+1$)-dimensional nonlinear evolution equation of four families of the solutions.

A Physical Explanation

The introduction of dispersion without introducing nonlinearity destroys the solitary wave as different Fourier harmonics start propagating at different group velocities. On the other hand, introducing nonlinearity without dispersion also prevents the formulation of solitary waves, because the pulse energy is frequently pumped into higher frequency models. However, if both dispersion and nonlinearity are present, solitary waves can be sustained. Similarity to dispersion, dissipation can also give rise to solitary wave when combined with nonlinearity. Hence it is more interesting to point out that the delicate balance between the nonlinearity effect and the dissipative effect gives rise to solitons solitary waves, that after a full interaction with others the solitons come back retaining their identities with the same speed and shape. The general ($2+1$)-dimensional nonlinear evolution equation has many solitary wave solutions. There is various type of traveling wave solutions that one of particular interest in solitary wave theory. For some special values of the physical parameters, we obtain the traveling wave solutions as follows:

The solitary wave solutions of kink type corresponding to $u_1(\xi)$ for the fixed values of the parameters $a_0 = 2, \mu = -0.5, \lambda = 1, a = 1, b = 1, A = 1.5, y = 0$ within $-3 \le x, t \le 3$ have presented in Fig. 1. Also Fig. 2 shows the solitary wave solutions of kink type corresponding to $u_5(\xi)$ with fixed parameters $a_0 = 1, \mu = -1.5, \lambda = 1.5, a = 1, b = 1, A = 0.5, y = 0$ within the interval $-3 \le x, t \le 3$. The exact periodic traveling wave solutions corresponding to $u_7(\xi)$ for the values of the parameter $a_0 = 2, \mu = 1.5, \lambda = 2, a = 0.5, b = 0.5, A = 0, y = 0$ within the interval $-3 \le x, t \le 3$ is shown in Fig. 3. Again, the exact periodic traveling wave solutions corresponding to $u_{10}(\xi)$ and $u_{12}(\xi)$ for the values $a_0 = 2, \mu = 1.5, \lambda = 2, a = 0.5, b = 0.5, A = 1, y = 0$ within the interval $-3 \le x, t \le 3$. The periodic wave solution is shown in Fig. 4 and Fig. 6 respectively. And for the fixed values of $a_0 = 2, \mu = 1.5, \lambda = 2, a = 0.5, b = 0.5, A = 5, y = 0$ within the interval $-3 \le x, t \le 3$ solutions of $u_{11}(\xi)$ also given the exact solitary wave solutions of periodic shape have presented in Fig. 5.

B Graphical Explanation

This sub-section represents the graphical representation of the general ($2+1$)-dimensional nonlinear evolution equation. By using mathematical software Maple 13, 3D and 2D plots of some achieved solutions have been shown in Fig. 1 to Fig. 6 to envisage the essential instrument of the original equations.

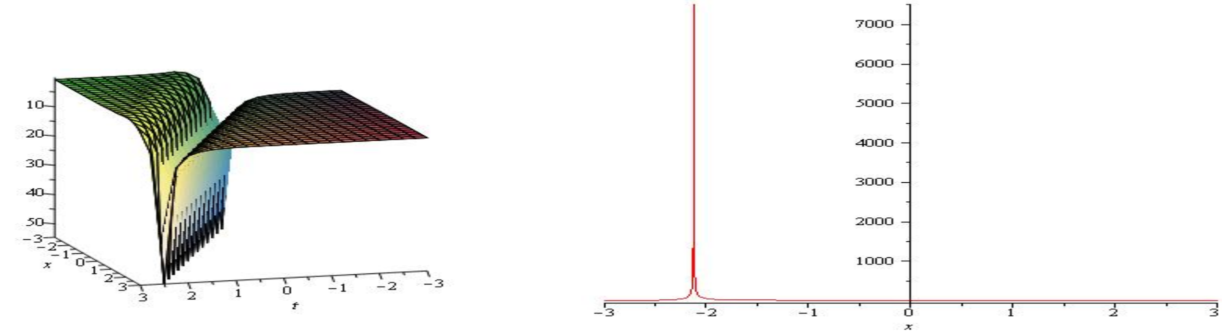

Fig. 1: Shape of $u_1(\xi)$, for $a_0 = 2, \mu = -0.5, \lambda = 1, a = 1, b = 1, A = 1.5, y = 0$ within $-3 \le x \le 3$ and $-3 \le t \le 3$. The top figure shows the 3D plot and the next figure shows the 2D plot for $t = 0$.

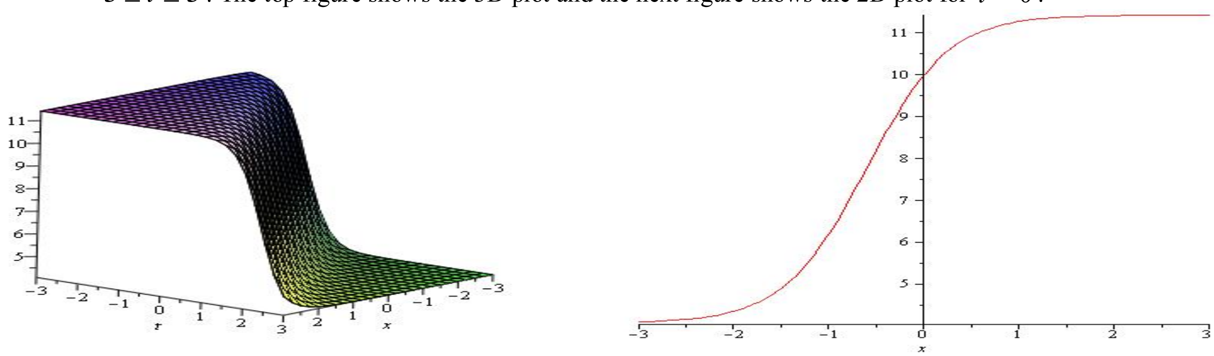

Fig. 2: Shape of $u_5(\xi)$, for $a_0 = 1, \mu = -1.5, \lambda = 1.5, a = 1, b = 1, A = 0.5, y = 0$ within $-3 \le x \le 3$ and $-3 \le t \le 3$. The top figure shows the 3D plot and the next figure shows the 2D plot for $t = 0$.

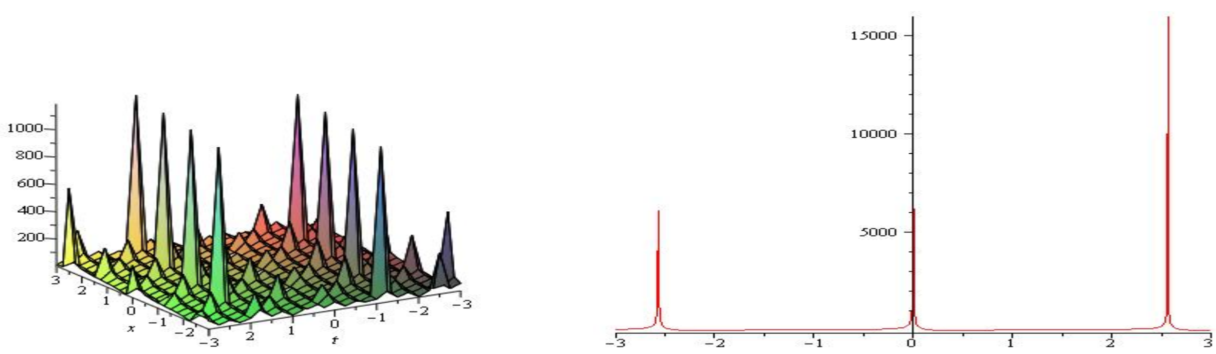

Fig. 3: Shape of $u_7(\xi)$, for $a_0 = 2, \mu = 1.5, \lambda = 2, a = 0.5, b = 0.5, A = 0, y = 0$ within $-3 \le x \le 3$ and $-3 \le t \le 3$. The top figure shows the 3D plot and the next figure shows the 2D plot for $t = 0$.

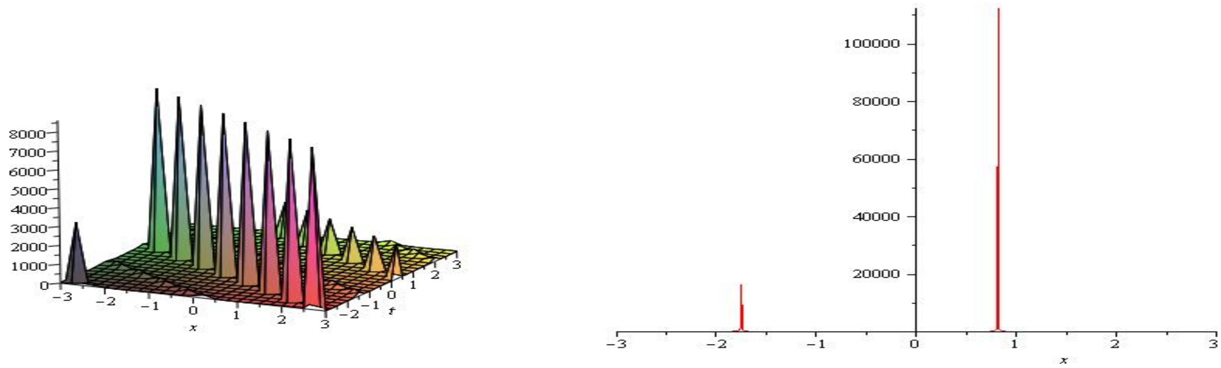

Fig. 4: Shape of $u_{10}(\xi)$, for $a_0 = 2, \mu = 1.5, \lambda = 2, a = 0.5, b = 0.5, A = 1, y = 0$ within $-3 \le x \le 3$ and $-3 \le t \le 3$. The top figure shows the 3D plot and the next figure shows the 2D plot for $t = 0$.

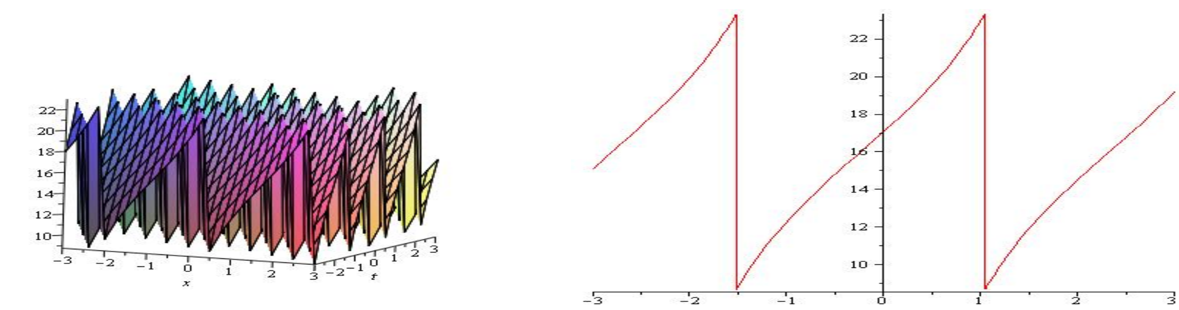

Fig. 5: Shape of $u_{11}(\xi)$, for $a_0 = 2, \mu = 1.5, \lambda = 2, a = 0.5, b = 0.5, A = 5, y = 0$ within $-3 \le x \le 3$ and $-3 \le t \le 3$. The top figure shows the 3D plot and the next figure shows the 2D plot for $t = 0$.

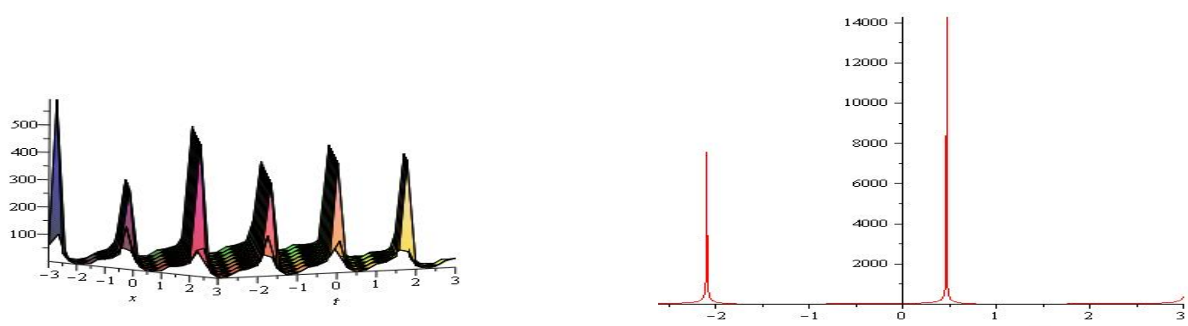

Fig. 6: Shape of $u_{12}(\xi)$, for $a_0 = 2, \mu = 1.5, \lambda = 2, a = 0.5, b = 0.5, A = 1, y = 0$ within $-3 \le x \le 3$ and $-3 \le t \le 3$. The top figure shows the 3D plot and the next figure shows the 2D plot for $t = 0$.

C Comparison with Existing Methods

Najafi *et al.* [16] examined exact solution of the general $(2+1)$-dimensional nonlinear evolution equation by using (G'/G)-expansion method and obtained 5 solution (see Appendix A). On the contrary by using the enhanced (G'/G)-expansion method we obtained 12 solutions. From our obtained solutions in this study, easily we can say that the enhanced (G'/G)-expansion method is very powerful, effective and convenient. The performance of this method is reliable, simple and gives many new solutions. On the other hand, it is notable to mention that the exact solutions of the general $(2+1)$-dimensional nonlinear evolution equation have been reached in this article using any commercial software, since the computations are very simple and easy. Similarly, for any nonlinear evolution equation it can be shown that the enhanced (G'/G)-expansion method is much powerful than other methods.

V. Conclusions

The basic goal of this work is to execute the enhanced (G'/G)-expansion method for precisely solving the nonlinear (2+1)-dimensional nonlinear evolution equation. Solutions of the (2+1)-dimensional nonlinear evolution equation are acquired via the enhanced (G'/G)-expansion method. The solutions are incorporated with two of explicit solutions namely, hyperbolic, and trigonometric functions, which are represented kink, periodic, and singular periodic wave behaviours for choosing the specific values of constants. This method blended with the symbolic computation package possesses an effective and simple mathematical tool for better understanding of the physical significance. The results also demonstrate that the free parameters of the functions have substantial effect on the wave behaviour, which might be used to visualize many new features that occur in different applied scientific areas.

APPENDIX-A

Najafi *et al.* [16] obtained exact solution of the general $(2+1)$-dimensional nonlinear evolution equation by using (G'/G)-expansion method. The obtained solutions are given below:

$$u(\xi)=\frac{6\sqrt{\lambda^2-4\mu}}{a+b}\left(\tanh\left(\frac{1}{2}\right)\sqrt{\lambda^2-4\mu}\,\xi-\lambda\right)+a_0$$

$$u(\xi)=-\frac{6\sqrt{4\mu-\lambda^2}}{a+b}\left(\tan\left(\frac{1}{2}\right)\sqrt{4\mu-\lambda^2}\,\xi+\lambda\right)+a_0$$

$$u(\xi)=\frac{6\sqrt{\lambda^2-4\mu}}{a+b}\left(\coth\left(\frac{1}{2}\right)\sqrt{\lambda^2-4\mu}\,\xi-\lambda\right)+a_0$$

$$u(\xi)=\frac{6\sqrt{4\mu-\lambda^2}}{a+b}\left(\cot\left(\frac{1}{2}\right)\sqrt{4\mu-\lambda^2}\,\xi-\lambda\right)+a_0$$

When $\lambda^2-4\mu=0$

$$u(\xi)=\frac{12}{a+b}\left(\frac{C_2}{C_1+C_2\xi}-\frac{\lambda}{2}\right)+a_0$$

where $\xi=x+y-(\lambda^2-4\mu)t$ and C_1,C_2,λ,μ are arbitrary constant.

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